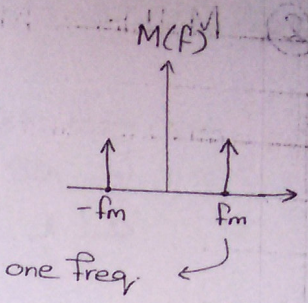
Summary of AM rules

① Single tone modulation

means $m(t)$ is only one freq. f_m

- $m(t) = A_m \cos(2\pi f_m t)$, $c(t) = A_c \cos(2\pi f_c t)$ ⟶



- $S(t) = A_c (1 + \underbrace{K_a . A_m}_{\mu} \cos(2\pi f_m t)) \cos(2\pi f_c t)$
- $\mu = \frac{A_m}{A_c} = K_a . A_m$.

$$S(t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{\frac{A_c \mu}{2}}_{\text{DSB}} \Big[\cos(2\pi (f_c - f_m) t + \cos(2\pi (f_c + f_m) t) \Big]$$

$\text{DSB} = m(t) * c(t)$

- Average power

$$P_{avg.} \longrightarrow R_L = 1 \Longrightarrow \frac{V_{Peak}^2}{2} = \frac{1}{T_o} \int_0^{T_o} g^2(t)\, dt$$

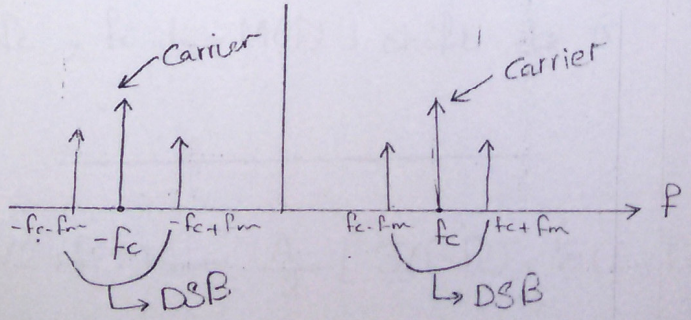
$$P_c = \frac{A_c^2}{2} \quad , \quad P_{SB} = \frac{A_c^2 \mu^2}{8} \quad , \quad P_{DSB} = \frac{A_c^2 \mu^2}{4}$$

$$P_t = P_c + P_{DSB} = \frac{A_c^2}{2} \left[1 + \frac{\mu^2}{2} \right]$$

$$\eta = \frac{P_{DSB}}{P_t} \% = \frac{A_c^2 \mu^2 / 4}{A_c^2/2 + A_c^2 \mu^2 / 4} \% = \frac{\mu^2}{2 + \mu^2} \%$$

- Peak Power $= \frac{V_{Peak}^2}{R_L} = I_{Peak}^2 . R_L$.

$$P_{C_{Peak}} = \frac{A_c^2}{R_L} \quad , \quad P_{DSB_{Peak}} = \frac{A_c^2 \mu^2}{2 R_L}$$

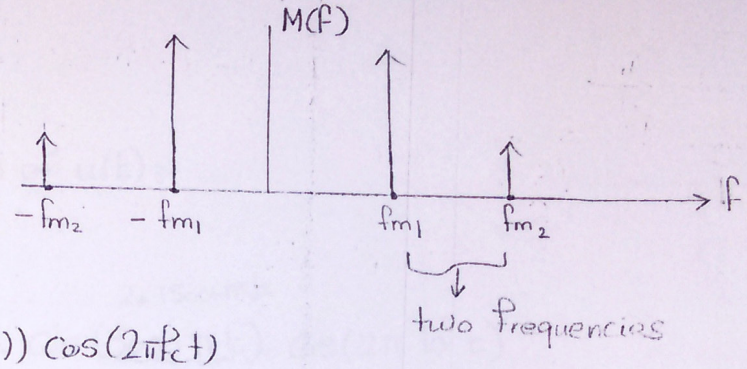


② Multitone Modulation

$$m(t) = A_{m_1}\cos(2\pi f_{m_1} t) + A_{m_2}\cos(2\pi f_{m_2} t)$$

سواء cos أو Sin

هنا الـ message أكثر من تردد فيسمى بالـ Multitone



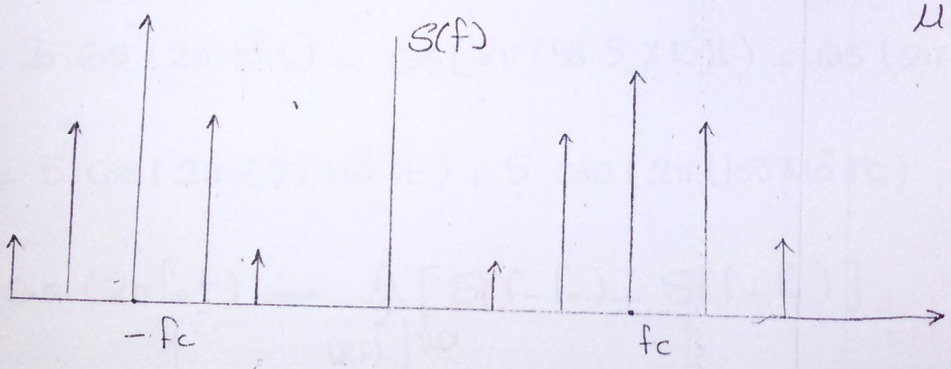
$$S(t) = A_c\,(1 + K_a \cdot m(t))\cos(2\pi f_c t)$$

$$S(t) = A_c\left[1 + \mu_1\cos(2\pi f_{m_1} t) + \mu_2\cos(2\pi f_{m_2} t)\right]\cos(2\pi f_c t)$$

- $\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$
  ↳ Total

- $P_{DSB} = \dfrac{A_c^2 \mu_t^2}{4}$ ← مثل

. كل باقي القوانين كما هي لكن نعوض بدل μ بالـ μ_t



نفس طريقة الرسم : ١) δ عند f_c و $-f_c$

٢) عمل shift لـ M(f) حول f_c و $-f_c$

Remember

$$A\cos(2\pi f_c t) \rightleftharpoons \frac{A}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$A\sin(2\pi f_c t) \rightleftharpoons \frac{A}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right].$$

Sheet #3 AM Sheet

D An AM signal has the form

$$u(t) = [20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)]\cdot\cos(2\pi f_c t)$$

where $f_c = 10^5$ Hz.

1. Sketch the voltage spectrum of $u(t)$.

$U(f)$? → draw

$$u(t) = 20\cos(2\pi f_c t) + 2\cos(\underbrace{3000\pi t}_{2\cdot 1500\cdot\pi\cdot t})\cdot\cos(2\pi\cdot 10^5 t)$$

$$+ 10\cos(6000\pi t)\cdot\cos(2\pi\cdot 10^5 t)$$

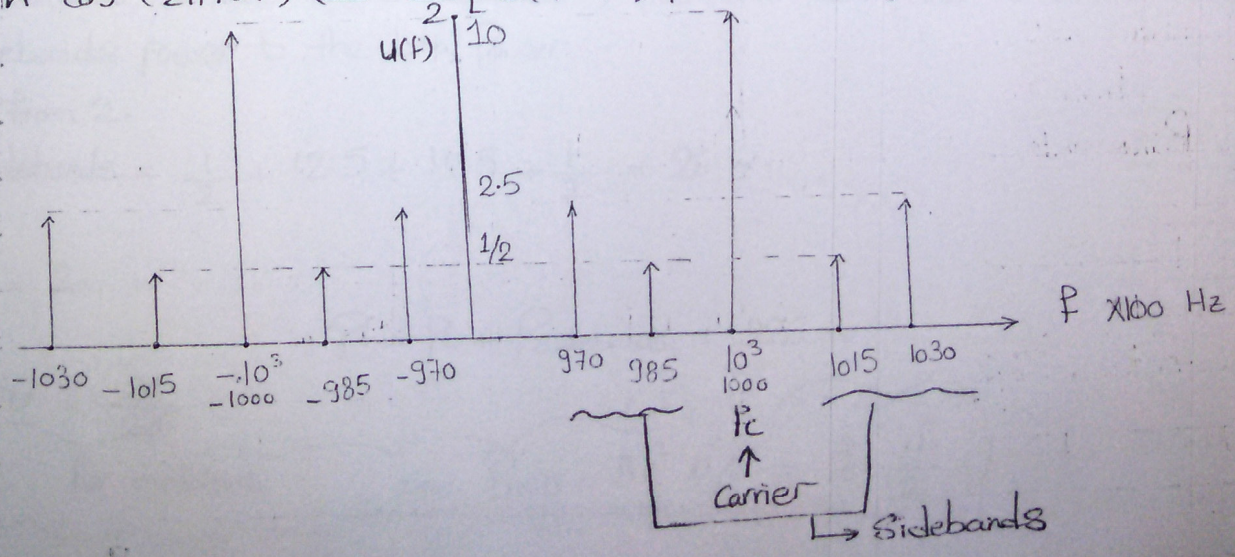
$$= 20\cos(2\pi f_c t) + \frac{2}{2}\left[\cos(2\pi(10^5 - 1500)t) + \cos(2\pi(10^5 + 1500)t)\right]$$

$$+ \frac{10}{2}\left[\cos(2\pi(\overset{f_c}{10^5} - 3000)t) + \cos(2\pi(\overset{f_c}{10^5} + 3000)t)\right]$$

$$= 20\cos(2\pi\cdot 10^5 t) + \cos(2\pi(98.5\times 10^3)t) + \cos(2\pi(101.5\times 10^3)t)$$

$$+ 5\cos(2\pi(97\times 10^3)t) + 5\cos(2\pi(103\times 10^3)t).$$

$$A\cos(2\pi f_0 t) \rightleftharpoons \frac{A}{2}[\delta(f - f_0) + \delta(f + f_0)]$$



2. Determine the Power in each of the frequency components.

Remember that for $A\cos\theta \rightarrow P_{avg.} = \frac{A^2}{2}$

هاشوف كل δ في الـ Spectrum الـ cosine تاخدها فين وأربع الـ Peak وأقسمها على اتنين.

- $f = 10^5 \rightarrow P_{avg.} = \frac{20^2}{2} = 200\ W$
- $f = \underset{(f_c-1500)}{985\times10^3}$ and $\underset{(f_c+1500)}{1015\times10^3} \rightarrow P_{avg.} = \frac{1}{2}\ W$
- $f = 97\times10^3$ and $103\times10^3 \rightarrow P_{avg.} = \frac{5^2}{2} = 12.5\ W$

3. Determine the modulation index

$$u(t) = 20\left[1 + \underbrace{0.1}_{\mu_1}\cos(3000\pi t) + \underbrace{0.5}_{\mu_2}\cos(6000\pi t)\right]\cos(2\pi f_c t)$$

(*) $\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$ for multitone $m(t)$

$= 0.509$

4. Determine the power in the sidebands, the total power and the ratio of the sidebands power to the total power.

from 2.

$P_{sidebands} = \frac{1}{2} + 12.5 + 12.5 + \frac{1}{2} = 26\ W.$

$P_c = 200\ W.$

$P_t = P_c + P_{sidebands} = 226\ W.$

$Ratio = \frac{26}{226}$

(*) Note: for multitone use $P_{DSB} = A_c^2 \cdot \mu_t^2/4 = P_c \cdot \frac{\mu_t^2}{2}$

(2) An AM signal is generated by modulating the carrier $f_c = 800$ kHz by the signal

$$m(t) = \sin(2000\pi t) + 5\cos(4000\pi t)$$

The AM signal

$$u(t) = 100\,[1 + m(t)]\cos(2\pi f_c t)$$

is fed to a 50 Ω load

1. Determine & sketch the spectrum of the AM signal.

$$u(t) = 100\left[1 + \underset{\mu_1}{1}\sin(2\pi(1000)t) + \underset{\mu_2}{5}\cos(2\pi(2000)t)\right]\cos(2\pi f_c t)$$

$$= 100\cos(2\pi f_c t) + \frac{100}{2}\left[\sin(2\pi(1000 - f_c)t) + \sin(2\pi(1000 + f_c)t)\right]$$

$$+ \frac{500}{2}\left[\cos(2\pi(2000 - f_c)t) + \cos(2\pi(2000 + f_c)t)\right]$$

$$= 100\cos(2\pi f_c t) - 50\sin(2\pi(f_c - 1000)t) + 50\sin(2\pi(f_c + 1000)t)$$

$$+ 250\cos(2\pi(f_c - 2000)t) + 250\cos(2\pi(f_c + 2000)t).$$

$$\sin x \cdot \cos y = \frac{1}{2}\left[\sin(x - y) + \sin(x + y)\right]$$

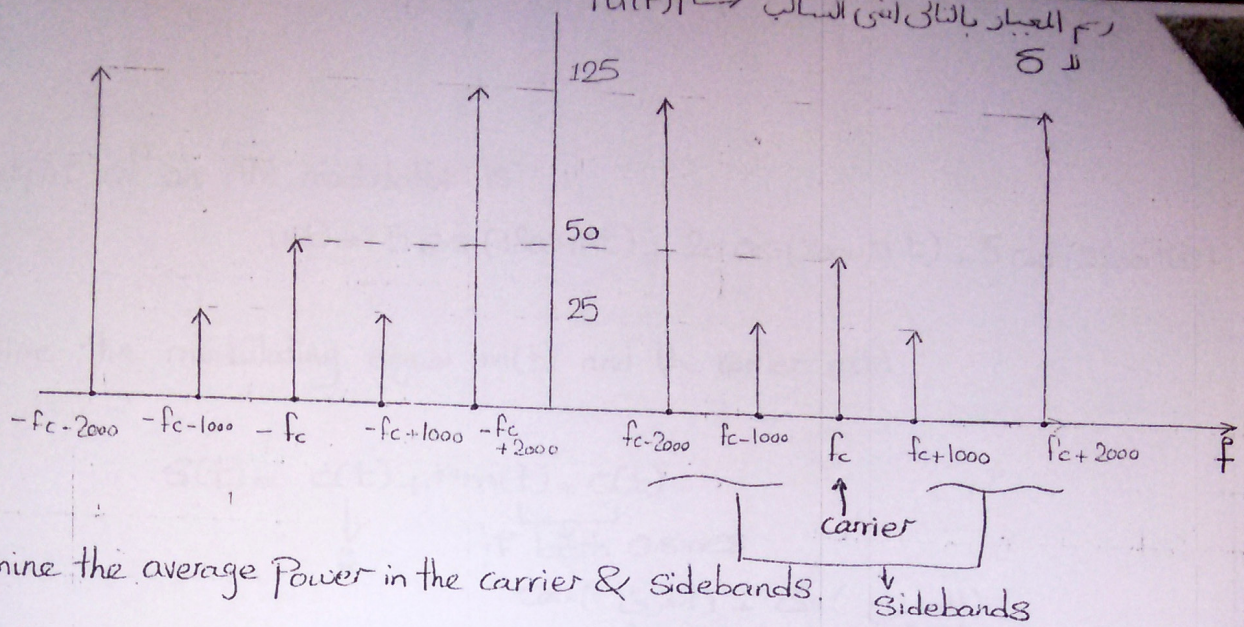
$$A\cos 2\pi f_0 t \rightleftharpoons \frac{A}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

$$A\sin 2\pi f_0 t \rightleftharpoons \frac{A}{2j}\left[\delta(f - f_c) - \delta(f + f_c)\right]$$

$$U(f) = 50\left[\delta(f - f_c) + \delta(f + f_c)\right] - \frac{25}{j}\left[\delta(f - (f_c + 1000)) - \delta(f + (f_c + 1000))\right]$$

$$+ \frac{25}{j}\left[\delta(f - (f_c + 1000)) - \delta(f + (f_c + 1000))\right] + 125\left[\delta(f - (f_c - 2000))\right.$$

$$\left. + \delta(f + (f_c - 2000))\right] + 125\left[\delta(f - (f_c + 2000)) + \delta(f + (f_c + 2000))\right].$$



2. Determine the average Power in the carrier & sidebands.

$$P_c = \frac{100^2}{2} = 5000.\,W$$

$$P_{sidebands} = \frac{50^2}{2} + \frac{250^2}{2} + \frac{50^2}{2} + \frac{250^2}{2} = 65000.\,W$$

3. What is the modulation index ?

$\mu_1 = 1$

$\mu_2 = 5$

$\mu_t = \sqrt{1+5^2} = 5.099.$

4. What is the Peak Power delivered to the load ?

③ The output of an AM modulator is

$$u(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t)$$

1. Determine the modulating signal $m(t)$ and the carrier $c(t)$.

$$s(t) = \underbrace{c(t)}_{f_c} + \underbrace{k_a m(t) * c(t)}_{\text{if both cosines}}$$

$\cos(\text{الفرق}) + \cos(\text{المجموع})$

$f_c - f_m$, $f_c + f_m$

$\therefore$ $20\cos(2000\pi t) \longrightarrow$ Carrier

$5\cos(1800\pi t) + 5\cos(2200\pi t) \longrightarrow k_a m(t) \cdot c(t)$

$$u(t) = 20\left[1 + \frac{1}{2}\cos(200\pi t)\right]\cos(2000\pi t)$$

$* \; 20\cos(2000\pi t) \cdot k_a m(t) = \frac{5}{2}\left[\cos(1800\pi t) + \cos(2200\pi t)\right]$

$\frac{1}{2}\cos(200\pi t)$ $\qquad$ $\frac{20 * A_m * k_a}{2} \xrightarrow{k_a A_m} A_m = 1/2$

$\therefore$ $m(t) = \cos(200\pi t)$

2. Determine the modulation index.

$$\mu = k_a \cdot A_m = \frac{1}{2}$$

3. Determine the ratio of the Power in the sidebands to the Power in the carrier.

$$P_c = \frac{20^2}{2} = 200\ W$$

$$P_{sideband} = \frac{A_c^2 \cdot \mu^2}{4} = \frac{(20 * 1/2)^2}{4} = 25\ W. \quad \text{or} \quad \frac{5^2}{2} + \frac{5^2}{2}$$

$$\therefore \frac{P_{sideband}}{P_c} = \frac{25}{200} = \frac{1}{8}.$$

When you're down and troubled
And you need some love care

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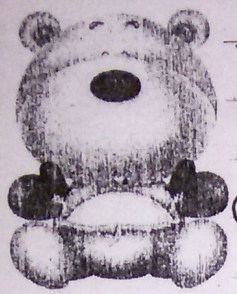


Sheet #3

① audio signal $m(t) = 15\cos(2\pi \cdot 1500 t) \rightarrow Am = 15 \quad fm = 1500\ Hz$

$c(t) = 60\cos(2\pi \cdot 100000 t) \rightarrow Ac = 60 \quad fc = 10^5\ Hz$

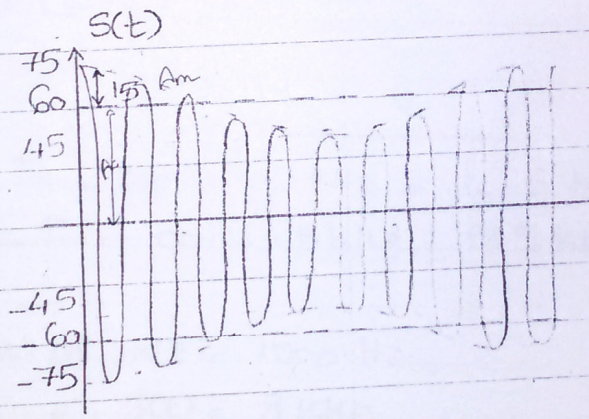
a) AM equation

$$S(t) = Ac\left(1 + \underbrace{Ka \cdot Am}_{\mu}\cos(2\pi fm t)\right)\cos(2\pi fc t)$$

$$\mu = \frac{Am}{Ac} = \frac{15}{60} = 0.25 \quad \text{or} \quad \mu = \frac{Amax - Amin}{Amax + Amin} = \frac{(Ac+Am)-(Ac-Am)}{(Ac+Am)+(Ac-Am)}$$

$$S(t) = 60 \cdot (1 + 0.25\cos(2\pi \cdot 1500 t))\cos(2\pi \cdot 100000 t)$$

b) Sketch the AM wave.



$\mu < 1 \rightarrow$ undermodulation

c) Modulation (index / factor) μ , Percent modulation

$$\mu = \frac{Amax - Amin}{Amax + Amin} = \frac{75 - 45}{75 + 45} = 0.25$$

$$\mu\% = 0.25 * 100 = 25\ \%$$

$c(t)(1 + k_a \cdot m(t))$

$c(t) + k_a \cdot m(t) \cdot c(t)$

For the memories of yesterday,
For the happiness of today...

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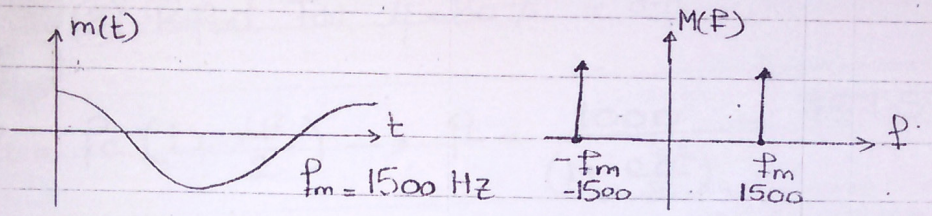
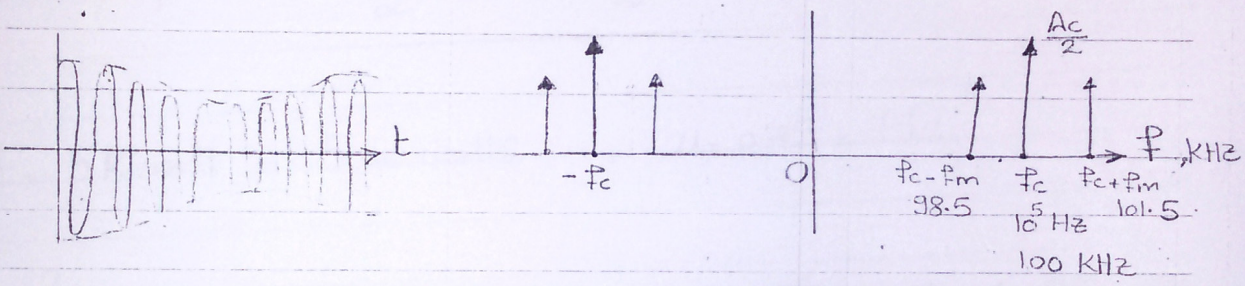
$M(f) \cdot \cos(2\pi f_c t)$

d) AM spectrum

$m(t) \longrightarrow M(f)$





B.W = $2f_m = 3$ KHz

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f) $f_u = f_c + f_m = 100K + 1.5\ KHz = 101.5\ KHz$

$f_L = f_c - f_m = 100k - 1.5\ KHz = 98.5\ KHz$

Note: B.W. of $m(t) = 1500$ Hz

" " $S(t) = 3$ KHz

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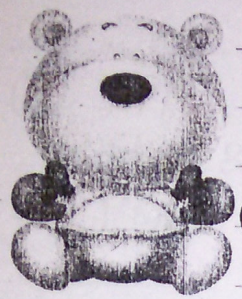
(2) $P_t = 1000$ w

$\mu\% = 100\% = 1$

* $P_c = ?$ ** P sideband = ?

* $P_t = P_c \cdot \left[1 + \frac{\mu^2}{2}\right] \rightarrow P_c = \frac{1000}{(1 + 1/2)} = 666.66$ watt

* $P_{DSB} = P_t - P_c = 1000 - 666.66 = 333.33$ watt



$P_{LSB} = P_{USB} = \frac{P_{DSB}}{2} = 166.67$ watts

③ Repeat for $\mu = 80\% = 0.8$

$P_t = P_c\left(1 + \frac{\mu^2}{2}\right) \rightarrow P_c = \frac{1000}{\left(1 + \frac{0.8^2}{2}\right)} = 757.57$ watts

$P_{LSB} = P_{USB} = \frac{P_{DSB}}{2} = \frac{1000 - 757.57}{2} = 121.21$ watts

④ $P_c = 5$ kwatt $= 5000$ watts $\quad \mu = 0.75 = 75\%$

a) $P_t = ?$

$P_t = P_c\left(1 + \frac{\mu^2}{2}\right) = 5000\left(1 + \frac{0.75^2}{2}\right) = 6406.25$ watts

b) P_{LSB} , P_{USB} , η

$P_{DSB} = P_t - P_c = 6406.25 - 5000 = 1406.25$ watts

- $P_{LSB} = P_{USB} = \frac{P_{DSB}}{2} = 703.125$ watts

- $\eta = \frac{P_{DSB}}{P_t}\% = \frac{1406.25}{6406.25}\% = 21.95\%$

c) Comment

نلاحظ أن الكفاءة منخفضة جداً وذلك لأن معظم الباور ضائعة في الـ Carrier الذي لا تحتوي على أي معلومة

⑤ $\mu\% = ?$ Percent modulation

$P_c = 8\ KW$

$P_{SB} = 2\ KW$

$P_{DSB} = 2 * 2KW = 4\ KW$

$P_t = P_c + P_{DSB} = 8 + 4 = 12\ KW$

$P_t = P_c\left(1 + \frac{\mu^2}{2}\right) \rightarrow 1 + \frac{\mu^2}{2} = \frac{P_t}{P_c} = \frac{12}{8}^{3}_{2}$

$\frac{\mu^2}{2} = \frac{3}{2} - 1 \rightarrow \boxed{\mu = 1}$

$\therefore \mu\% = 100\%$

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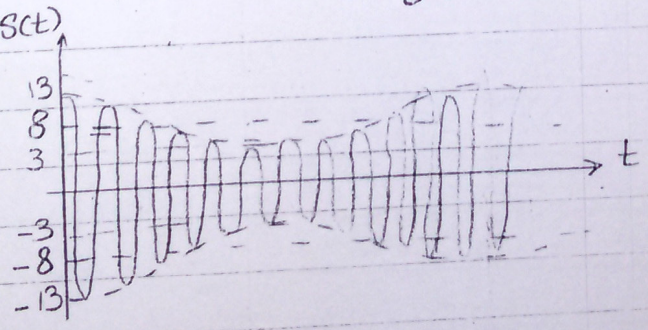
⑥ $m(t) = 5\cos(2\pi 500t) \rightarrow A_m = 5$, $f_m = 500$

$P_c = 32$ watt

$K_a = 0.125 = \frac{1}{A_c} \rightarrow A_c = \frac{1}{0.125} = 8$ $\boxed{A_c = 8}$

a) Sketch AM wave

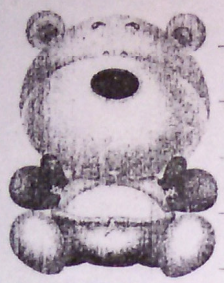
$\mu = K_a \cdot A_m = \frac{1}{8} * 5 = 0.625$



b) Write AM equation

$S(t) = A_c(1 + K_a \cdot m(t)) \cdot \cos(2\pi f_c t)$

$S(t) = 8(1 + 0.625\cos(2\pi \cdot 500t))\cos(2\pi f_c t)$



c) $P_t = ?$ $\eta = ?$ Comment

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$$

$$= 32\left(1 + \frac{0.625^2}{2}\right) = 38.25 \text{ watts}$$

$$P_{DSB} = P_t - P_c = 38.25 - 32 = 6.25 \text{ watts}$$

$$\eta = \frac{P_{DSB}}{P_t} \% = 16.3\ \%$$

we notice that η is low due to the unuseful Power wasted in the carrier, $\eta = \frac{P_{useful}}{P_t} \% = \frac{P_{useful}}{P_c + P_{useful}} \%$

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d) B.W of $m(t) = f_m = 500$ Hz

B.W. of AM $= 2f_m = 1000$ Hz
